Statistics and Causality

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Introduction

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Contents

- *The Simpson paradox*
- *Approaches to causality in experimental settings*
- *Approaches to causality in nonexperimental setting*
- *Applications*
- *Conclusions*

Simpson paradox 1

Table 1. *Evaluation of a treatment*

	treatment		
success	yes ($X = 1$)	no ($X = 0$)	total
yes ($Y = 1$)	500	600	1100
no ($Y = 0$)	500	400	900
	1000	1000	2000

Note. After Novick (1980). Numbers are fictitious.

Simpson paradox 2

Tabelle 1. *Evaluation of treatment conditional on gender*

A. Males ($W = 1$)

success	treatment: yes ($X = 1$)	treatment: no ($X = 0$)	total
yes ($Y = 1$)	300	75	375
no ($Y = 0$)	450	175	625
	750	250	1000

Simpson paradox 3

B. females ($W = 0$)

success	treatment: yes ($X = 1$)	treatment: no ($X = 0$)	total
yes ($Y = 1$)	200	525	725
no ($Y = 0$)	50	225	275
	250	750	1000

Note:. After Novick (1980). The numbers are fictitious.

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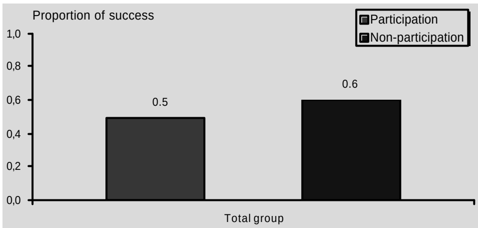
Simpson paradox 4



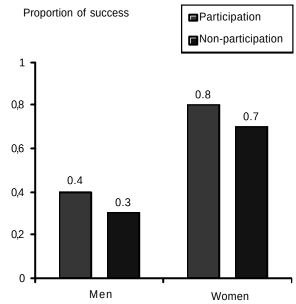


Figure 2. **Histogram of proportion of success, conditional on gender**

Approaches to causality in experimental settings

The role of selecting units within treatments (simplified illustration 1)

Table 1. *Illustration of a homogeneous population*

Person	Y_1 Potential outcome under treatment	Y_0 Potential outcome under control	$Y_0 - Y_1$ Individual causal effect
u_1	110	100	10
u_2	110	100	10
u_3	110	100	10
u_4	110	100	10
u_5	110	100	10
u_6	110	100	10
u_7	110	100	10
u_8	110	100	10
Mean	**110**	**100**	**10**

Approaches to causality in experimental settings

The role of selecting units within treatments (simplified illustration 2)

Table 1. *Example illustrating individual and average causal effects*

(a) Experiment with non-comparable groups

Person	Y_1 Potential outcome under treatment	Y_0 Potential outcome under control	$Y_1 - Y_0$ Individual causal effect
u_1	**82**	68	14
u_2	89	**81**	8
u_3	**101**	89	12
u_4	**108**	102	6
u_5	118	**112**	6
u_6	**131**	119	12
u_7	139	**131**	8
u_8	152	**138**	14
Mean	**105.5**	**115.5**	10

Note. The red numbers are selected.

Approaches to causality in experimental settings

The role of selecting units within treatments (simplified illustration 3)

(b) Experiment with comparable groups

Person	Y_1 Potential outcome under treatment	Y_1 Potential outcome under control	$Y_1 - Y_0$ Individual causal effect
u_1	82	**68**	14
u_2	**89**	81	8
u_3	**101**	89	12
u_4	108	**102**	6
u_5	**118**	112	6
u_6	131	**119**	12
u_7	139	**131**	8
u_8	**152**	138	14
Mean	**115**	**105**	10

Note. The red numbers are selected.

Approaches to causality in experimental settings

The role of sampling units within treatments (Rubin)

Person	Y_1 Potential outcome under treatment	Y_0 Potential outcome under control	$Y_1 - Y_0$ Individual causal effect	$P(X = 1 \mid Y_0, Y_1)$ treatment probability in experiment 1	$P(X = 1 \mid Y_0, Y_1)$ treatment probability in experiment 2
u_1	82	68	14	1/2	8/9
u_2	89	81	8	1/2	7/9
u_3	101	89	12	1/2	6/9
u_4	108	102	6	1/2	5/9
u_5	118	112	6	1/2	4/9
u_6	131	119	12	1/2	3/9
u_7	139	131	8	1/2	2/9
u_8	152	138	14	1/2	1/9
Mean	115	105	10		

Approaches to causality in experimental settings (Steyer et al., 2000)

Table 1. *Individual causal effects*.

Person	$E(Y \mid X = 1, U = u)$ Expected outcome	$E(Y \mid X = 0, U = u)$ Expected outcome	$E(Y \mid X = 1, U = u) - E(Y \mid X = 0, U = u)$ Individual causal effect
u_1	82	68	14
u_2	89	81	8
u_3	101	89	12
u_4	108	102	6
u_5	118	112	6
u_6	131	119	12
u_7	139	131	8
u_8	152	138	14
	Individual causal laws		

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Approaches to causality in experimental settings (Steyer et al., 2000)

Table 1. *Individual causal effects and treatment probabilities.*

Person		$E(Y \mid X = 1, U = u)$ Expected outcome	$E(Y \mid X = 0, U = u)$ Expected outcome	$E(Y \mid X = 1, U = u) - E(Y \mid X = 0, U = u)$ Individual causal effect
u_1		82	68	14
u_2		89	81	8
u_3		101	89	12
u_4		108	102	6
u_5		118	112	6
u_6		131	119	12
u_7		139	131	8
u_8		152	138	14

Individual causal laws

Table 1. *Individual and average causal effects*.

Person	$P(U=u)$ sampling probability	$E(Y \mid X = 1, U = u)$ Expected outcome	$E(Y \mid X = 0, U = u)$ Expected outcome	$E(Y \mid X = 1, U = u) - E(Y \mid X = 0, U = u)$ Individual causal effect
u_1	1/8	82	68	14
u_2	1/8	89	81	8
u_3	1/8	101	89	12
u_4	1/8	108	102	6
u_5	1/8	118	112	6
u_6	1/8	131	119	12
u_7	1/8	139	131	8
u_8	1/8	152	138	14

Individual causal laws

Average causal laws:

$$\sum_u E(Y \mid X = x, U = u) \cdot P(U = u)$$

Table 1. *Individual causal effects and equal treatment probabilities*.

Person	$P(U=u)$ sampling probability	$E(Y \mid X=1, U=u)$ Expected outcome	$E(Y \mid X=0, U=u)$ Expected outcome	$E(Y \mid X=1, U=u) - E(Y \mid X=0, U=u)$ Individual causal effect	$P(X=1 \mid U=u)$ treatment probability in experiment 1
u_1	1/8	82	68	14	1/2
u_2	1/8	89	81	8	1/2
u_3	1/8	101	89	12	1/2
u_4	1/8	108	102	6	1/2
u_5	1/8	118	112	6	1/2
u_6	1/8	131	119	12	1/2
u_7	1/8	139	131	8	1/2
u_8	1/8	152	138	14	1/2
		Individual causal laws			Good design

Average causal laws:

$$\sum_u E(Y \mid X = x, U = u) \cdot P(U = u)$$

Good design implies: $E(Y \mid X = x)$ reflect average causal laws

Table 1. *Individual causal effects, equal and unequal treatment probabilities*.

$$E(Y \mid X = x) := \sum_u E(Y \mid X = x, U = u) \cdot P(U = u \mid X = x)$$

Person	$P(U=u)$ sampling probability	$E(Y \mid X=1, U=u)$ Expected outcome	$E(Y \mid X=0, U=u)$ Expected outcome	$E(Y \mid X=1, U=u) - E(Y \mid X=0, U=u)$ Individual causal effect	$P(X=1 \mid U=u)$ treatment probability in experiment 1	$P(X=1 \mid U=u)$ treatment probability in experiment 2
u_1	1/8	82	68	14	1/2	8/9
u_2	1/8	89	81	8	1/2	7/9
u_3	1/8	101	89	12	1/2	6/9
u_4	1/8	108	102	6	1/2	5/9
u_5	1/8	118	112	6	1/2	4/9
u_6	1/8	131	119	12	1/2	3/9
u_7	1/8	139	131	8	1/2	2/9
u_8	1/8	152	138	14	1/2	1/9
		Individual causal laws			Good design	Bad design

Average causal laws:

$$\sum_u E(Y \mid X = x, U = u) \cdot P(U = u)$$

Good design implies: $E(Y \mid X = x)$ reflect average causal laws

Bad design implies: $E(Y \mid X = x)$ do not reflect average causal laws

There are conditions under which, for each value x of X:

$$E(Y\,|\,X = x) := \sum_u E(Y\,|\,X = x, U = u) \cdot P(U = u\,|\,X = x)$$

is equal to

$$\sum_u E(Y\,|\,X = x, U = u) \cdot P(U = u).$$

For each value x of X:

(1) $P(U = u \mid X = x) = P(U = u) \quad \forall\, u$

or

(2) $E(Y \mid X = x, U = u) = E(Y \mid X = x) \quad \forall\, u$

Unconfoundedness of the regression $E(Y \mid X)$

For each value x of X:

(1) $P(U = u \mid X = x) = P(U = u)$,

or

(2) $E(Y \mid X = x, U = u) = E(Y \mid X = x)$

Unconfoundedness of the regression $E(Y \mid X)$

Unconfoundedness is equivalent to:

For each $W := f(U)$:

$$E(Y\,|\,X = x) := \sum_w E(Y\,|\,X = x, W = w) \cdot P(W = w)$$

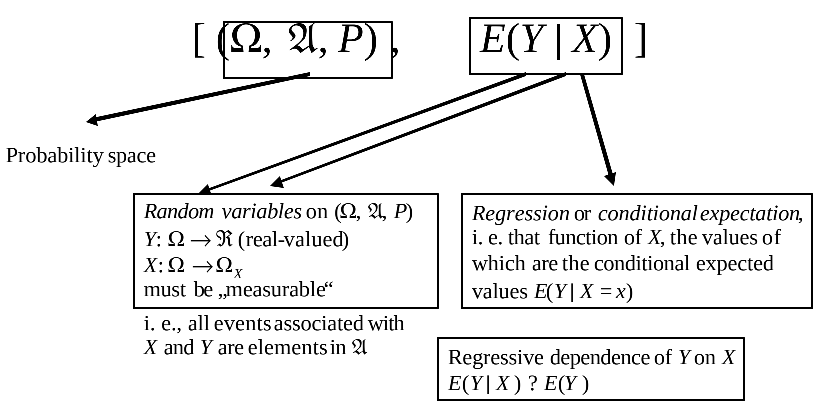
for each value x of X.

Table 2. An example in which the treatment regression $E(Y \mid X)$ is unconfounded and causally unbiased but none of the other causality criteria discussed in this paper hold

Observational-unit variable U	$P(U=u)$	W (gender)	$P(X=x_1 \mid U=u)$	$E(Y \mid X=x_1, U=u)$	$P(X=x_2 \mid U=u)$	$E(Y \mid X=x_2, U=u)$	$P(X=x_3 \mid U=u)$	$E(Y \mid X=x_3, U=u)$
u_1	1/8	*m*	1/2	82	1/10	105	4/10	110
u_2	1/8	*m*	1/2	89	1/10	105	4/10	110
u_3	1/8	*m*	1/2	101	2/10	105	3/10	110
u_4	1/8	*m*	1/2	108	2/10	105	3/10	110
u_5	1/8	*f*	1/2	118	3/10	105	2/10	110
u_6	1/8	*f*	1/2	131	3/10	105	2/10	110
u_7	1/8	*f*	1/2	139	4/10	105	1/10	110
u_8	1/8	*f*	1/2	152	4/10	105	1/10	110

Note: The (unconditional) probabilities for the three treatments are $P(X=x_1) = 1/2$, $P(X=x_2) = P(X=x_3) = 1/4$.

Approaches to causality in experimental settings 1 (Steyer et al., 1984, 1992)



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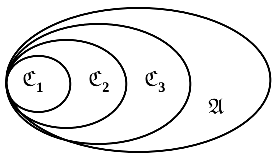
Approaches to causality in experimental settings 2 (Steyer et al., 1984, 1992)

$$[\,(\Omega, \mathfrak{A}, P),\ E(Y \mid X)\ ,\ (\mathfrak{C}_t,\ t \in T),\ \mathfrak{D}\,]$$

same as before

Monotonically nondecreasing family of σ-algebras $\mathfrak{C}_t \subset \mathfrak{A}$

$\mathfrak{D} \subset \mathfrak{A}$, a sub-σ-algebra of $\mathfrak{A}$.



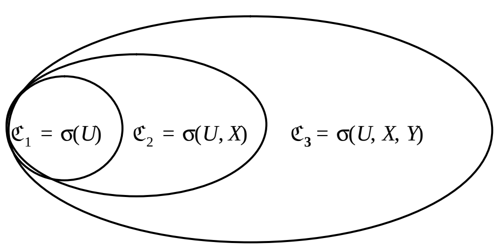
used to define preorderedness relation between events and random variables.
[Random variables generate σ-algebras $\subset \mathfrak{A}$.]

used to define „potential confounders" W (random variables). Their generated σ-algebra is a subset of $\mathfrak{D}$.

Pre-orderedness
$W \rightarrow X \rightarrow Y$

Approaches to causality in experimental settings 3 (Steyer et al., 1984, 1992)

$$[\,(\Omega, \mathfrak{A}, P),\ E(Y \mid X)\ ,\ (\mathfrak{C}_t,\ t \in T),\ \mathfrak{D}\,]$$



$\mathfrak{D} = \mathfrak{C}_1 = \sigma(U)$
Potential confounders W: measurable with respect to $\mathfrak{D}$

Pre-orderedness
$U \rightarrow X \rightarrow Y$

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Approaches to causality in experimental settings 4
(Steyer et al., 1984, 1992) Causality conditions

Strict Causality

$E(Y \mid X, W) = E(Y \mid X)$ for each potential confounder W

Strong Causality

$E(Y \mid X, W) = E(Y \mid X) + f(W)$ for each potential confounder W

Weak Causality (= Unconfoundedness)

If W is a potential confounder, then, for P^X-almost every value x of X:

$$E(Y \mid X = x) = ?\, E(Y \mid X = x, W = w)\, P^W(dw)$$

i.e., if W is discrete:

$$E(Y \mid X = x) = \sum_w E(Y \mid X = x,\ W = w)\ P(W = w)$$

Sufficient conditions for Weak Causality (Steyer, 1992)

1. Stochastic independence of X and $\mathfrak{D}$ implies Weak Causality. [If $\mathfrak{D}$ is defined to be generated by U, the random variable, the values of which are the observational units drawn from the population, then this independence can be deliberately created via random assignment of units to treatment conditions.]

2. Both, Strict and Strong Causality Conditions imply Weak Causality.

Applications

- Experimental design techniques such as randomization, conditional randomization etc.
- Data analysis techniques such as
  - Nonorthogonal Analysis of Variance
  - Analysis of Covariance
  - Computation of causal effects in structural equation models
  - Tests of confounding
  - Data mining for causal dependencies

  ….

Nonorthogonal Analysis of Variance

Table 1. Example for a nonorthogonal analysis of variance design

	Need for therapy						
	strong		medium		weak		total
Treatment	$Z = z_1$		$Z = z_2$		$Z = z_3$		
1 $X = x_1$	120	(40)	110	(20)	60	(6)	(66)
2 $X = x_2$	100	(14)	100	(80)	100	(14)	(108)
3 $X = x_3$	80	(6)	90	(20)	140	(40)	(66)
total		(60)		(120)		(60)	(240)

Note. True cell means and, in parentheses, cell frequencies.

Conclusions

- The mathematical structure of causal stochastic dependencies is now well-known
- The theory of stochastic causality helps in deciding between competing strategies for data analysis
- The theory also leads to new ways of data analysis
- Many statistical problems in these data analyses are not yet solved

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